Figurate Numbers and Forward Differences

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To find the *forward difference* of a list of numbers, we subtract each number from the next one in the list. For example, if we start with this list:

1 5 7 10 11 15 20

we first subtract 1 from 5, which gives 4. We write the 4 underneath the list, in between the 1 and the 5, like this:

1		5		7		10		11		15		20
	4											

Next we subtract the 5 from 7 which gives us 2. We can write the 2 in between the 5 and the 7, like this:

1		5		7		10		11		15		20
	4		2									

If we keep going and write the difference of each pair of numbers in between them, we get this:

1		5		7		10		11		15		20
	4		2		3		1		4		5	

Find the forward difference of each list of numbers. The first difference has been filled in for you.

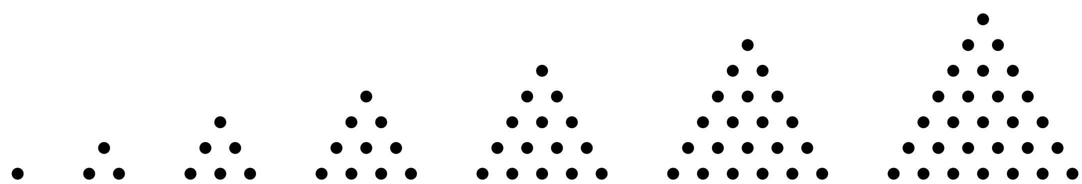
1	4	6	9	13	14	18
3						

1	3	5	7	9	11	13
2						

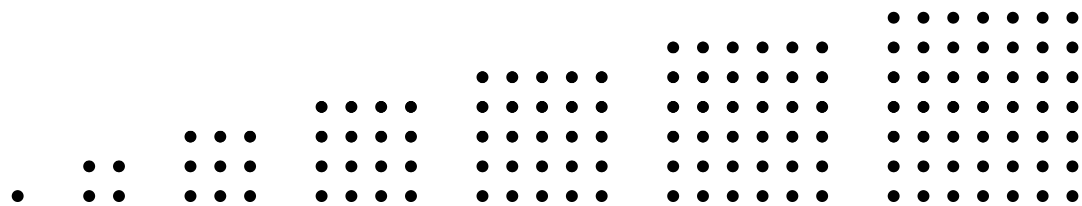
1	2	4	8	16	32	64
1						

1	2	3	5	8	13	21
1						

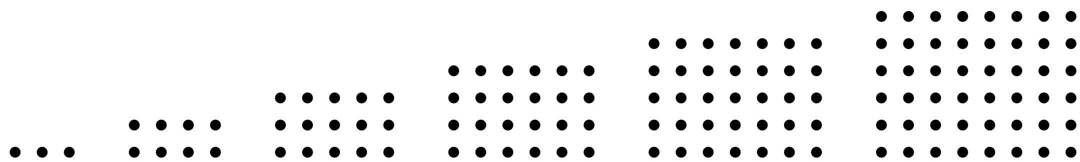
Write down the number of dots in each triangle. Then find the forward difference of the list of numbers that you get.



Write down the number of dots in each square. Then find the forward difference.



Write down the number of dots in each rectangle. Then find the forward difference.



Questions for further reflection:

1. Taking the forward difference of a list of numbers produces another list of numbers. What happens if we take the forward difference again? And again? And so on? Try it on some of the example lists above.

2. Can you use your observations to predict the next triangle number? The next square number? The next $n \times (n + 2)$ rectangle number?